EQUIVALENCE RELATIONS FOR A LASER RESONATOR WITH A LENS-LIKE MEDIUM — APPROXIMATION RESULTING FROM THE HUYGENS-FRESNEL PRINCIPLE

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Using the Fresnel scalar formulation of the Huygens principle, the problem of determining the electromagnetic field distribution at the mirrors of an optical resonator filled with a medium displaying a parabolic variation of the index of refraction has been reduced to a simpler question of the so-called, passive equivalent resonator. The values of the equivalent resonator parameters are identical with those obtained by the Kogelnik approach, based on the ray matrix formulation of geometrical optics.

1. Introduction

Certain properties of the laser resonators filled with a lens-like medium as well the influence of such medium on the laser radiation have recently been considered in literature ([1], [2], [3], [4], [5], [6]). Some aspects of the forenamed resonators have been explained ([1], [5]) by the Kogelnik theory [7], based on the ray matrix approach of geometrical optics. Following the increasing interest in the resonators with lens-like media, an attempt is made in this paper to present the theory of such a resonator with a medium of which the index of refraction varies in radial direction (in the plane perpendicular to the resonator axis).

A parabolic radial variation of the index of refraction of a medium is assumed. Such an assumption comprises a broad variety of functions describing the actual distributions of the index of refraction. The system of integral equations, presented in this paper, describing the distribution of the electromagnetic field at the resonator mirrors, is obtained from the Fresnel scalar formulation of the Huygens principle ([8], [9]). This is permissible in cases where the dimensions of the mirrors are large in terms of wavelength, and when the field is very nearly a transverse electromagnetic one (the waves of TE and TM types) and it is uniformly polarized in one direction. Taking into consideration

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a relatively small change of the index of refraction of the medium and basing on the results of the work [10], it may be assumed that the application of the Huygens-Fresnel principle is justified in the cases in question.

2. *System of integral equations and equivalence parameters of a resonator with lens-like medium*

An optical resonator formed by two spherical (or flat) mirrors M_1 and M_2 spaced at a distance d is considered, as shown in Fig. 1. The radii of curvature of the mirrors are R_1 and R_2, respectively. The mirrors are assumed to be circular, and of equal diameters $2a$. All the resonator dimensions are assumed large against the wavelength. It is, additionally, assumed that

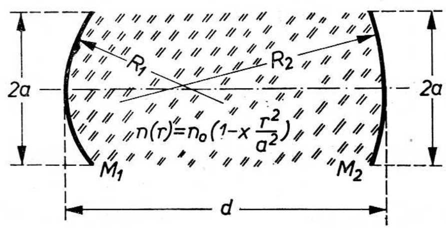

Fig. 1. Geometry of an optical resonator filled with a medium having a parabolic radial variation of the index of refraction

$$a \ll d, \tag{1}$$

which is typical for the optical resonators.

The medium filling all the space between the mirrors is assumed to be determined by the parabolic radial variation of the index of refraction

$$n(r) = n_0\left(1-x\frac{r^2}{a^2}\right), \tag{2}$$

where:
r — distance from the resonator axis, x — characteristic parameter of the medium.

The characteristic medium parameter x is negative ($x < 0$) for the medium with defocusing properties and positive ($x > 0$) for the focusing one. It is assumed that the characteristic medium parameter x is small against unity, *i.e.*,

$$|x| \ll 1. \tag{3}$$

Introducing the definition of the resonator mode in a steady state [9]:

$$\psi_q^{(j)} = [\gamma^{(j)}]^q \psi^{(j)}, \quad (j = 1 \text{ or } 2) \tag{4}$$

and using the Fresnel scalar formulation of the Huygens principle the system of integral equations describing the distribution of electromagnetic field at mirrors M_1 and M_2 of