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System of Integral Equations and Equivalence Parameters of an Optical Resonator with Inhomogeneous Medium — Approximation Resulting from Quasi-Geometric Optics

by

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Summary. Using the quasi-geometric optics formulation for media with inhomogeneous index of refraction, the problem of determining the electromagnetic field distribution at the mirrors of an optical resonator filled with a medium displaying a parabolic variation of the index of refraction has been reduced to a simpler problem of an equivalent empty optical resonator. From the derived system of integral equations the equivalent generalized parameters of the considered resonator are determined. The presented approach allows to compare the diffraction losses, the resonant conditions and the mode patterns of a resonator in question with respect to an equivalent one.

Introduction. The majority of papers on the subject of laser resonators deal with the theory of empty optical resonators or resonators filled with optically homogeneous media. However, the active media of lasers may, for various reasons (e.g. [1—7]), be optically inhomogeneous. An inhomogeneous medium inserted between the resonator mirrors affects the properties of the optical resonator, i.e. it changes the mode patterns, diffraction losses and conditions of resonance.

Following the increasing interest in resonators, filled with inhomogeneous media, the demand arises to solve the problem of the field distribution, diffraction losses and resonant frequency for such resonators. The exact solution of Maxwell's equation with appropriate boundary conditions, even when available, is too complicated to be used for determining the information about the resonator in question. In the literature, there is the Kogelnik's well-known approach [8] based on the ray matrix formulation of geometric optics and the imaging rules obtained with the use of formalism of Fresnel diffraction theory.

The purpose of this paper is to present the theory of an optical resonator with a medium displaying a parabolic variation of the index of refraction developed from the formulation of quasi-geometric optics for inhomogeneous media proposed by Eichmann [9]. The choice of such a form of the index of refraction is justified since the parabolic variation of the index comprises a broad variety of functions describing the actual distribution of the index of refraction. Eichmann's formulation of quasi-geometric optics for inhomogeneous media is analogous to Feynman's approach [10] to quantum mechanics.

Quasi-geometric-optics approach for media of inhomogeneous index of refraction. Assume that at any surface σ_1 of an inhomogeneous medium, the distribution of the electromagnetic field $\Psi(x_1, y_1, z_1)$ is known. The distribution $\psi(x, y, z_1+\epsilon)$, where ϵ is a small distance in the direction of coordinate z can be found from the relation [9, 10]:

$$\psi(x, y, z_1+\epsilon)=\iint_{\sigma_1} \exp[-ikS(x, y, x_1, y_1, \epsilon)]\,\psi(x_1, y_1, z_1)\frac{dx_1}{A}\frac{dy_1}{A}, \tag{1}$$

where A is an ϵ-dependent normalization constant and k is the wave number in the medium. The optical path length

$$S(x, y, x_1, y_1, \epsilon)=\int_{z_1}^{z_1+\varepsilon} L(x, y, \dot{x}, \dot{y}, z)\,dz \tag{2}$$

is the integral of the optical lagrangian

$$L(x, y, \dot{x}, \dot{y}, z)=n(x, y, z)(1+\dot{x}^2+\dot{y}^2)^{\frac{1}{2}} \tag{3}$$

taken along the path which makes the optical path length (2) an extremum; $n(x, y, z)$ is the inhomogeneous index of refraction of the medium. The dots represent differentation with respect to z. Eq. (1) is true in the limit $\epsilon\to 0$.

The integral (1) has been presented for the first time by Feynman [10] as the expression of the Huygens' principle for matter waves in his approach to quantum mechanics. Basing on Eq. (1) Eichmann [9] proposed the formulation of quasi-geometric optics for media with inhomogeneous index of refraction.

At any arbitrary distance z, $\psi(x, y, z)$ can be obtained by iterating Eq. (1) by distances ϵ along the z direction until z is reached, resulting in

$$\psi(x, y, z)=\iint_{\sigma_1} \mathcal{K}(x, y, z, x_1, y_1, z_1)\,\psi(x_1, y_1, z_1)\,dx_1\,dy_1, \tag{4}$$

where the kernel of integral (4)

$$\mathcal{K}(x, y, z, x_1, y_1, z_1)=\lim_{\varepsilon\to 0}\frac{1}{A}\frac{1}{A}\int_{-\infty}^{\infty}\cdots\int_{-\infty}^{\infty}\exp\left[-ik\sum_{j=1}^{l} S(x_{j+1}, y_{j+1}, x_j, y_j, \epsilon\right]\times \tag{5}$$
$$\times\frac{dx_2}{A}\cdot\frac{dx_3}{A}\cdots\frac{dx_l}{A}\cdot\frac{dy_2}{A}\cdot\frac{dy_3}{A}\cdots\frac{dy_l}{A}=$$
$$=\int \exp[-ikS(x, y, z, x_1, y_1, z_1)\,Dx(z)\cdot Dy(z)$$

is the continuous-path integral [11]. In Eq. (5) it is assumed that x_{l+1}, y_{l+1} are the coordinates of the surface at which the disturbance $\psi(x, y, z)$ is searched for. The integral equation (4) gives the field distribution $\psi(x, y, z)$ at any surface $\sigma(x, y, z)$ if distribution $\psi(x_1, y_1, z_1)$ at a surface σ_1 is known.